ABSTRACT. This work describes derivation of a control model for electrode melting and heat and mass transfer from the electrode to the workpiece in gas metal arc welding (GMAW). Specifically, a model is developed which allows electrode speed and welding speed to be calculated for given values of voltage and torch-to-base metal distance, as a function of the desired heat and mass input to the weldment. Heat input is given on a per unit weld length basis, and mass input is given in terms of transverse cross-sectional area added to the weld bead (termed reinforcement). The relationship to prior work is discussed.

The model was demonstrated using a computer-controlled welding machine and a proportional-integral (PI) controller receiving input from a digital filter. The difference between model-calculated welding current and measured current is used as controller feedback. The model is calibrated for use with carbon steel welding wire and base plate with Ar-CO₂ shielding gas. Although the system is intended for application during spray transfer of molten metal from the electrode to the weld pool, satisfactory performance is also achieved during globular and streaming transfer. Data are presented showing steady-state and transient performance, as well as resistance to external disturbances.

Introduction

In recent years, there has been a significant growth in the use of mechanized welding systems as cost-effective substitutes for shielded metal arc welding. Perhaps the most popular process for such substitution is gas metal arc welding (GMAW) and its variants, especially for arc welding applications using robots. Yet the GMAW process is subject to the same fundamental limitations as virtually all other arc welding processes. Control of the process is limited to those factors to which machine builders are accustomed, such as electrode speed, welding speed, current, and voltage. The factors that the welding engineer would like to control, such as reinforcement area, weld heat input, macro- and microstructure, and mechanical and physical properties of the weld, are not directly controllable except in isolated cases.

This work is an effort to independently control two of the above factors, namely the reinforcement (defined as the transverse cross-sectional area of the metal added during welding) and the heat input to the base metal per unit length of weld bead. This paper describes the development of a relatively simple, steady-state model of the gas metal arc welding process that supports direct, independent control of heat and mass input to the weld. The application of this model, including software and a computer-controlled GMAW machine design, is discussed.

Background

The gas metal arc welding process employs a consumable electrode passing through a copper alloy contact tube — Fig. 1. Electrical current, imposed on the electrode by a voltage drop between the contact tube and the metal to be welded (workpiece), supports an arc between the electrode end and the workpiece. The electrode is melted by internal resistive power and heat transferred from the arc. Droplets of molten metal are detached and transferred from the electrode to the weld pool by a combination of gravitational, Lorentz, surface tension, and plasma forces (Ref. 1). Heat is transferred to the base metal directly
Prior efforts at modeling the process have concentrated on the electrode melting rate. For electrode positive welding, electrode melting rate \( m' \) has been determined empirically by Lesnewich (Ref. 2) for mild steel to be of the form

\[
m' = (C_1 + C_2 A)I + \frac{C_3 LI^2}{A}
\]

where \( A \) is electrode cross-sectional area, \( I \) is current, \( L \) is electrical extension, and \( C_1, C_2, C_3 \), and \( i \) are constants. More recently, Waszink and Van den Heuvel (Ref. 3) derived an expression of similar form for spray transfer of mild steel

\[
m' = C_4 I + \frac{C_5 LI^2}{A}
\]

where \( C_4 \) and \( C_5 \) are constants.

In both cases, electrode melting rate is given as a function of electrode extension \( L \), which is the distance from the point of current entry into the electrode to the electrode end at the arc. Electrode extension is not easily measured since the effective point of current entry into the electrode is not known. In addition, locating the electrode end is not a trivial problem due to the presence of the arc. Thus, using such expressions for real-time control of the process would be difficult. (It should be noted that the above authors did not represent their work as being suitable for real-time control.) Nonetheless their work is of considerable importance. The relationships between their work and the present paper are discussed in Appendix A. The expression for electrode melting rate needs to be derived in terms of the contact tube to base metal distance and voltage drop, parameters that are easily measured.

The GMAW process may be performed with the electrode at a positive or a negative potential with respect to the workpiece, using either a constant current, power, or voltage power supply, with either pulsed or steady electrode speed and/or welding current. However, the most common operating conditions are electrode positive, using a constant voltage power supply, with constant electrode speed and current. This work is limited to such conditions.

The process operating region is shown in Fig. 2A as a function of electrode speed and welding speed and in Fig. 2B as a function of heat input and reinforcement, for specified values of open circuit voltage and contact tube-to-workpiece distance. Lines of constant value of heat and mass input to the weld have been superimposed on the operating region. Of interest is the fact that changes in either electrode speed or welding speed will change both the heat and mass inputs to the weld. Independent adjustments of heat and mass inputs require combined changes of both electrode speed and weld speed in some unique ratio.
The present approach uses a process model to calculate the parameters required to obtain desired values of heat and mass input to the weld. Although it would be best to develop a model that would give exact solutions, the model is an approximation which is fitted to the process periodically, based on an error signal fed into a controller. The controller output updates the model to correspond more closely to the process conditions. There are several desired characteristics of such an approach. One, the steady-state process output should closely approximate the desired output. Two, the process should be stable, yet responsive in the presence of transients. This includes changes in the desired input, as well as during starting of the weld. Three, the process should be tolerant of external disturbances. This work addresses these items.

Model Derivation

Portions of the model derivation have been presented in Refs. 4–7. The complete model derivation is shown in Appendix A. In summary, the power required to melt the electrode is given by

\[ IV_w + \eta^I IV_a = \frac{\delta SH_m D^2 \pi}{4R} \]  

where \( V_w \) is voltage drop along the electrode, \( \eta^I \) is efficiency of heat transfer from the arc to the electrode, \( V_a \) is voltage drop across the arc, \( \delta \) is density, \( S \) is electrode feed speed, \( D \) is electrode diameter, \( R \) is welding speed, and \( H_m \) is total heat required to melt a unit volume of material, given as

\[ H_m = \int_{T_0}^{T_s} C_d dT + H_f \]  

where \( C_d \) is specific heat, \( T_0 \) and \( T_s \) are ambient and superheat temperatures, and \( H_f \) is heat of fusion.

Heat input to the base metal per unit length of weld (\( H \)) is given by (Ref. 8)

\[ H = \frac{E \eta}{R} \]  

where \( E \) is secondary circuit voltage drop, \( \eta \) is heat transfer efficiency from the process to the base metal, and \( R \) is welding speed.

Weld bead reinforcement (\( G \)), defined as the transverse cross-sectional area of the deposited metal, is given by

\[ G = \frac{SD \eta}{R} \]  

where \( E \) is secondary circuit voltage, \( \eta \) is heat transfer efficiency from the process to the base metal, and \( R \) is welding speed.

Output voltage of the power supply (\( E \)) is given by

\[ E = E_o + nI = H \frac{dl}{dt} \]  

where \( E_o \) is open circuit voltage of the supply (i.e., at zero load), \( n \) is the effective slope of the supply (i.e., the combined effects of power supply slope and secondary circuit resistance), \( H \) is secondary circuit inductance, and \( t \) is time. Current is maintained by the power supply consistent with the contact tube-to-workpiece resistance, which is determined by the electrode extension and arc resistance, and the contact tube-to-workpiece voltage. Power consumed by the process is approximated as the sum of that consumed by resistive heating of the electrode and that consumed in the arc as

\[ IE = IV_w + IV_a \]  

Finally, Ohm's law is used

\[ IV_a = I^2R_a \]

where \( R_a \) is arc resistance and both sides of Equation 9 are multiplied by the current. In normal GMAW, current is controlled by changing electrode speed, and
reinforcement is controlled by changing welding speed. Thus, heat and mass inputs to the weld are not controlled independently. Yet, such independence can be obtained by solving Equations 3–9, appropriately.

It can be shown (see Appendix A) that

\[ S = \frac{E_n \eta_4 G I + n \eta_4 G I^2}{\pi D^2 H} \]  

and

\[ R = \frac{S \pi D^2}{4G} \]  

where \( S \) is electrode speed, \( R \) is welding speed, \( G \) is reinforcement, and \( H \) is heat input per length of weld. Also, current \( I \) is given by

\[ I = K'_0 + K'_1 (CT) + K'_2 E_n + K'_3 S \]  

Equations 10–12 may be solved iteratively. Limits for these equations may be deduced by considering that in Equation 8, as \( V_w \) goes to zero, arc length approaches the contact tube-to-workpiece distance. In this case, the arc transfers to the contact tube and the process is uncontrollable. The other limiting case is for \( V_a \) to go to zero, and in this case, the electrode stubs into the weld pool.

Equations 10, 11, and 12 were solved and the solutions plotted as a function of the ratio of reinforcement to heat input \((G/H)\). A typical plot is shown in Fig. 3. The current \( I \) is valid for any values of reinforcement and heat input which give \( G/H \) ratios in the range of 0 to 50 \( \text{mm}^3/\text{J} \). The electrode speed \( S \) and welding speed \( R \) are only for a heat input of 1250 \( \text{J/mm} \), although solutions may be found for other heat inputs in a similar manner.

**Sensing**

Electrical current is measured during welding by obtaining the voltage drop across a calibrated shunt placed in the secondary current loop. The resulting voltage signal is noisy, corresponding to a current variation too great to allow direct measurement of voltage to be useful for controlling the process. The solution to this problem is to filter the signal, in this case by a digital, linear difference technique.

The shunt voltage \( V_s \) is measured in sets of 30 values taken at the rate of 3600 per second. The last 20 of these values...
are processed by a simple finite impulse response (FIR) filter (Ref. 9)

\[ Y = \sum_{i=0}^{N} h_i X_i \]  
(13)

where \( h_i = 0.05 \). Thus, \( Y \) is the average of 20 data points \( X_i \). The first ten values are discarded to remove any artifacts associated with the relay switching used for multiplexing. At approximately one-third second intervals, the value from the finite impulse filter \( Y \) is processed using a simple infinite impulse response (IIR) filter (Ref. 9)

\[ Z = C' Y_o + \sum_{i=2}^{N} b_i Z_i \]  
(14)

where \( C' = 0.5 \) and \( b_i = 0.25 \). Thus, \( Z \), the measured current, is a weighted, moving average of past \( Z \) values and the present \( Y \) value. The results of this simple technique may be seen in Fig. 4 where a sample of measured current is shown with and without filtering. The current signal is also filtered using a digital, first-order, low-pass Butterworth filter (Ref. 9) having a 1-Hz cut-off frequency, shown for comparison purposes. It may be noted that a hardware low-pass filter should be placed in front of the digital filter if concern exists regarding aliasing errors.

### Welding System Controller

In the process control scheme, the difference between measured current (filtered) and calculated current is used as the error signal input \( e(t) \) to a proportional-integral (PI) controller (Ref. 10) which calculates a change in the parameter \( CT \) corresponding to the contact tube-to-workpiece distance in Equation 12

\[ \Delta(CT) = K_p e(t) \]

where \( K_p \) is the proportional gain and \( K_i \) is the integral gain. Integral error is calculated using Simpson's rule (Ref. 11). Thus, the value of \( CT \) is continuously adjusted to reduce the difference between measured and calculated current (but the actual contact tube-to-workpiece distance is not changed). The maximum change in \( CT \) is limited to ±10 mm at any time step. This allows welding with globular metal transfer. The corresponding control system block diagram is shown in Fig. 5.

#### Welding Hardware

The gas metal arc welding system developed for this work consists of a computer controlled welding head, positioning table, operator panel, and a welding power supply. Two power supplies have been used. These are a Linde SVA-300, which is a conventional 300-A constant voltage supply, and a Philips 450-A transistor-regulated supply operated in a constant voltage mode. In either case, the system described in this work connects to the contactor and secondary (welding) circuits of the power supply. No other connections are required. A
Fig. 7 — Measured current and voltage as a function of time for a weld (T22) made using a good set of controller gains. Time increases from right to left.

Diagram of the system is shown in Fig. 6. In operation, desired values of weld bead reinforcement (fill) and heat input per length of weld are set on the operator panel in units of mm² and J/mm, respectively. The system then controls electrode speed and welding speed, based on model solutions and current feedback, to give the desired values.

Software

Software for the computer is written in the HPL language (Ref. 12). The program operates in two major loops. The first loop monitors status of the operator panel prior to welding and displays desired heat input and reinforcement area values. Provisions are made to jog the weld table and electrode for setup. The second loop is entered upon actuation of the weld start/stop switch. In the second loop the power supply contacter is activated and the electrode feed and positioner table motors are driven at speeds calculated by the process model. Current is monitored and adjustments made to bring the current to the model solution, while welding speed is simultaneously adjusted to maintain the correct reinforcement. Desired heat input and reinforcement values may be changed during welding. Releasing the weld start/stop switch causes a downslope to zero electrode speed and welding speed along with deactivation of the power supply contactor. The first loop is then reentered.

Experimental Studies

An initial series of 35 welds was made for model calibration using three contact tube-to-workpiece distances, three open...
Controller Gains

Digital PI controllers may be tuned in the same manner as analog PI controllers. In this study, the sampling frequency is relatively high. Following Åström and Wittenmark (Ref. 10), using the transient response method of Ziegler and Nichols, the controller gains were estimated to be in the ranges of $K_p = 0.02$ to $0.4$ and $K_i = 0.02$ to $4.0$. In this work, values in these ranges were used as starting points and further tuning was done empirically. Additional tuning was necessary because the sampling frequency in this work is moderately low and the estimated gain ranges are large. Nonetheless, the controller gains are in the estimated ranges found by the transient response method.

Welds were made (using the Philips power supply) to evaluate effects of changing proportional and integral gains in Equation 15. For example, Fig. 7 shows current and voltage measurements taken with a HP 7132A strip chart recorder during a weld (T22) made with a heat input of 1600 J/mm (41 kJ/in.), reinforcement area of 40.4 mm², $K_p = 0.07$ and $K_i = 0.05$ and changes in $C_T$ limited to ±10 mm. Figure 7 shows a stable process start-up with spray transfer of metal. In similar experiments, gains for use with a Linde SVI power supply were found to be $K_p = 0.1$ and $K_i = 0.15$ with changes in $C_T$ not limited.

Figure 8A–8I illustrates more detail the behavior of the controller during weld T22. In Fig. 8A, weld current is shown, taken at a digitizing rate of 250 Hz. Data were taken by a separate data acquisition system (Hewlett-Packard Model 9000 Series 200 computer with an internal analog-to-digital converter). In Fig. 8B, welding current is shown as measured (filtered) by the welding system computer, and the current calculated by the model is shown in Fig. 8C. The error signal in Fig. 8D is the difference between current values shown in the two previous figures. The resulting controller output (CT) is shown in Fig. 8E. CT rapidly (< 3 s) approaches its steady-state value. This result is similar, rapid attainment of nominal steady-state values for electrode speed (S) and welding speed (R), Fig. 8F and G. Weld heat input per length of weld is shown in Fig. 8H, and calculated reinforcement is shown in Fig. 8I. Reinforcement is controlled by ensuring that the ratio of electrode speed to weld speed satisfies Equation 11. Consequently, in all welds the plot of G vs. t is similar, and the only variations are due to round-off errors.

Steady-State Performance

In order to evaluate the steady-state error of the control model, a series of 30 welds was made, three each at reinforcement areas of 15, 20, 25,...,60 mm². The respective heat inputs per length of weld were set to achieve spray transfer of molten metal to the weld pool. Heat input levels within a set of three welds were identical. Reinforcement areas were measured by sectioning, macroetching, and using a digitizer and area-measuring algorithm on macrophotographs of approximately 3X magnification. Heat inputs were measured using a liquid-nitrogen calorimeter technique described elsewhere (Refs. 13, 14). Measured reinforcement is plotted as a function of desired reinforcement in Fig. 9A;
measured heat input per length of weld is plotted as a function of desired values in Fig. 9B. In both cases, measured values are very close to desired values, and the behavior is very linear.

**Transient Performance**

Although this work is mainly concerned with the development of a control model, some examples of controller dynamic performance are also presented. Three transient events have been examined in this work. The first of these is the start of the process. As described above, the controller does not function during the first second of welding. Provided parameters have been selected to obtain spray transfer of molten metal to the weld pool, the start of the process, at least with the Philips transistorized power supply used, is completed before the controller activates. A resulting process start is seen in Fig. 4. This behavior is essentially equal to a conventional GMAW start. The use of a computer data-acquisition system to obtain welding current values during the first several seconds of welding illustrates the rapid rate at which the electrode melting process approaches equilibrium. In Fig. 4, it is seen that current is very close to the steady-state level within less than about 0.1 s. (The apparently slow approach to steady state shown in Fig. 7 is due to the fact that strip chart recorders generally function as low-pass filters. Indeed it does not require much filtering to introduce error into the data as may be seen by the effect of the 1-Hz low-pass Butterworth filter in Fig. 4, where it has apparently required about 0.5 s for the current to reach nominal steady state.) The controller used in this work does not function for the first three program iterations, i.e., the first second of welding. Thus, the rapid attainment of nominal steady-state current during start-up in weld T22 is due to the fast response of the transistorized power supply. The open-loop process is highly damped and stable over most of the operating range, presumably due to the strong coupling between the electrical and thermal aspects of the process. The behavior of the process for times greater than about 1 s is dominated by the characteristics of the controller. It is important in application of a feedback controller to the process that it remain stable. The present controller results in stable dynamics and damping is adequate.

The second transient event studied involved step changes in desired heat input while maintaining reinforcement constant. In Fig. 10, heat input per length of weld is plotted as a function of time for a weld in which the initial heat input was set at 1600 J/mm with reinforcement held at 40.4 mm². About 7 s into the weld the heat input was changed stepwise to 1400 J/mm (36 kj/in.) and then returned to about 1600 J/mm at 17 s into the weld. There is slight overshoot in the heat input value, followed by rapid decay to the new value for both changes. This behavior is representative of a slightly underdamped second-order system. The response of the system is good, requiring about 3 s to pass the transients. The amount of overshoot is small enough to be acceptable.

The third transient, representative of some extreme disturbance, involved extinguishing the arc briefly. Fig. 11A shows current plotted as a function of time for Weld T45. At about 3.6 s into the weld a wire was inserted into the arc, shorting it. The disturbance lasted approximately 1 s. The resulting error signal (e(t)) is plotted as a function of time in Fig. 11B. Although this is a major transient event, the controller is sufficiently robust to overcome the disturbance rapidly.

**Conclusions**

This work presents derivation and evaluation of a steady-state model of the
GMAW process, suitable for feedback control. The model is based on physical consideration of the process; simplifications have been made to allow timely computation. A simple feedback control scheme is used to demonstrate model application. At least two aspects of the control scheme are unusual and deserve comment. First, a steady-state model solution is used to control a dynamic process. Second, a multivariable process is controlled using a single feedback loop.

Three phenomena occur in the GMAW electrode melting process. These are electrical current flow, heat flow, and mass transfer. Of the three, the flow of electrical current dominates the behavior of the process, for it is current that generates heat and produces the major forces resulting in mass transfer. The time constant associated with changes in electrical current is much smaller than those associated with heat and mass transfer. But even so, heat and mass transfer occur at sufficiently high rates that their time constants can be effectively ignored. In addition, the process is stable during open loop operation. Thus, we are able to use a steady-state model solution for control purposes, even though this is not normally considered good practice.

The opportunity to use a single feedback loop for control results from domination of the process by the flow of electrical current. If we select the correct current, we will obtain the corresponding correct heat transfer. The functional form of the model contributes to the low steady-state error, and allows us to obtain the correct mass transfer.

In welds made with the control model during this work, various droplet transfer modes have been encountered. Satisfactory operation has been obtained during globular, spray, and streaming transfer. This allows the operating range to be shown in terms of heat input and reinforcement. Figure 2B presents such an operating range for carbon steel wire and base metal, Ar-0.5% O₂ shield gas, and certain other conditions such as voltage, contact tube-to-workpiece distance, and electrode diameter which may affect the position of the operating range in the domain.

It should be noted that our ability to take advantage of unique aspects of the GMAW process to simplify the control scheme does not imply that a similar approach can be used successfully for other processes.

Therefore, in summary are the following:

1) A steady-state model of GMAW electrode melting and workpiece heat and mass inputs has been successfully applied to the problem of independently controlling weld heat and mass inputs.

2) A simple proportional-integral controller has been developed, and controller gains determined to allow stable operation; a digital signal filtering routine is included.

3) A computer-controlled GMAW machine has been developed which is suitable for use as a front end to either a conventional constant-voltage or a transistor-regulated power supply.

Acknowledgments

This work was supported by the U.S. Department of Energy, Office of Basic Energy Sciences, under DOE Field Office, Idaho, Contract No. DE-AC07-76ID01570. Appreciation is expressed to U. S. Wallace for technical assistance.

Appendix A

Mathematical Basis for a Model of Weld Heat and Mass Input Control

Consider the electrode volume element (of cross-sectional area A and length dx) shown in Fig. 1. Applying conservation of energy to that volume element with sign convention based on energy transport into (or generation within) the element being positive, gives:

\[
\frac{dU}{dt} = \frac{\Delta Q}{\Delta t} - \frac{\Delta W}{\Delta t}
\]  
(A1)

where \( U \) is internal energy of the element, \( t \) is time, \( Q \) is heat, \( W \) is work, and \( t \) is time. Since \( \frac{\Delta W}{\Delta t} \) is zero, we need only solve for two terms of (A1)

\[
\frac{dU}{dt} = \delta C_p(T) A dx \frac{dT}{dt}
\]  
(A2)

and

\[
\frac{\Delta Q}{\Delta t} = (Q_{x+dx} - Q_x) + 
\left( \frac{I^2 \rho(T) dx}{A} \right) - 
\left( D \pi d x \sigma T^4 \right)
\]  
(A3)

where \( \delta \) is density, \( C_p \) is specific heat, \( T \) is temperature, \( t \) is time, \( x \) is location, \( I \) is current, \( \rho \) is electrical resistivity, \( D \) is electrode diameter, \( \varepsilon \) is emissivity, and \( \sigma \) is the Stefan-Boltzmann constant.

Substituting Equations A2 and A3 into A1 and expanding \( Q_{x+dx} \) in a Taylor series in \( x \) (Ref. 11), we now have

\[
\delta C_p A \frac{dT}{dt} = \frac{\partial Q_x}{\partial x} + \frac{I^2}{A} \rho(T) dx - D \pi d x \sigma T^4
\]  
(A4)

Dividing by \( dx \) and taking the limit as \( dx \to 0 \)

\[
\delta C_p A \frac{dT}{dt} = \frac{\partial Q_x}{\partial x} + \frac{I^2}{A} \rho(T) dx - D \pi d x \sigma T^4
\]  
(A5)

Fourier's Law states

\[
Q_x = -k A \frac{dT}{dx}
\]

where \( k \) is thermal conductivity. Thus, dividing by \( A \)

\[
\frac{\delta C_p A}{A} \frac{dT}{dt} = \frac{-\partial (k \frac{dT}{dx})}{\partial x} + \frac{I^2}{A^2} \rho(T) - D \pi d x \sigma T^4
\]  
(A-6)

Using the chain rule,

\[
\frac{dT}{dt} = \frac{dx}{dt} \frac{dT}{dx}
\]

But

\[
\frac{dx}{dt} = S
\]

so

\[
\frac{dT}{dt} = S \frac{dT}{dx}
\]  
(A7)

where \( S \) is electrode speed. Substituting Equation A7 in Equation A6, we have

\[
\delta C_p S \frac{dT}{dt} = -\frac{\partial (k \frac{dT}{dx})}{\partial x} + \frac{I^2}{A^2} \rho(T) - D \pi d x \sigma T^4
\]
we can calculate the amount of internal energy which the electrode must contain at the point where it just melts. This is given simply by

\[ A \delta S \int_{T_m}^{T_e} (C_p(T) \, dt + H_f) \]

where \( H_f \) is the heat of fusion.

Summing these components gives

\[ A \delta S \int_{T_m}^{T_e} (C_p(T) \, dt + H_f) = \]

\[ \eta \cdot IV_w \]

(A12)

which states that, at steady state, the power required to just melt the electrode is the sum of the Joule heating contribution and the power conducted from the anode spot to the solid portion of the electrode. (This result will be used later.)

Now consider the heat input (H) to the workpiece per unit length of weld given by

\[ H = \frac{E_l n}{R} \]

(A13)

where \( E_l \) is the open circuit voltage of the supply (i.e., at zero load), \( n \) is the effective slope of the supply (i.e., the combined effects of power supply slope and secondary circuit resistance), \( H \) is the power supply secondary circuit inductance, and \( t \) is time.

Current is maintained by the power supply consistent with the contact tube-to-base-metal resistance, which is determined by the electrode extension and

### Table 2 — Equations 10 and A-23 — I and S Predictions

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Measured</th>
<th>Calculated</th>
<th>Measured</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>192</td>
<td>195.2</td>
<td>183.8</td>
<td>186.2</td>
</tr>
<tr>
<td>2</td>
<td>186</td>
<td>190.7</td>
<td>171.7</td>
<td>175.0</td>
</tr>
<tr>
<td>3</td>
<td>185</td>
<td>180.7</td>
<td>158.4</td>
<td>155.6</td>
</tr>
<tr>
<td>4</td>
<td>165</td>
<td>161.9</td>
<td>121.1</td>
<td>119.3</td>
</tr>
<tr>
<td>5</td>
<td>210</td>
<td>215.7</td>
<td>175.3</td>
<td>178.9</td>
</tr>
<tr>
<td>6</td>
<td>213</td>
<td>206.9</td>
<td>157.0</td>
<td>153.6</td>
</tr>
<tr>
<td>7</td>
<td>193</td>
<td>202.6</td>
<td>136.3</td>
<td>143.5</td>
</tr>
<tr>
<td>8</td>
<td>237</td>
<td>239.6</td>
<td>176.8</td>
<td>178.2</td>
</tr>
<tr>
<td>9</td>
<td>248</td>
<td>243.3</td>
<td>197.3</td>
<td>194.6</td>
</tr>
<tr>
<td>10</td>
<td>253</td>
<td>246.5</td>
<td>213.5</td>
<td>209.6</td>
</tr>
<tr>
<td>11</td>
<td>227</td>
<td>237.5</td>
<td>164.0</td>
<td>169.5</td>
</tr>
<tr>
<td>12</td>
<td>180</td>
<td>189.7</td>
<td>183.8</td>
<td>191.3</td>
</tr>
<tr>
<td>13</td>
<td>175</td>
<td>183.6</td>
<td>171.7</td>
<td>178.3</td>
</tr>
<tr>
<td>14</td>
<td>172</td>
<td>159.3</td>
<td>154.8</td>
<td>144.9</td>
</tr>
<tr>
<td>15</td>
<td>153</td>
<td>159.9</td>
<td>124.1</td>
<td>125.5</td>
</tr>
<tr>
<td>16</td>
<td>201</td>
<td>192.4</td>
<td>159.2</td>
<td>153.9</td>
</tr>
<tr>
<td>17</td>
<td>205</td>
<td>211.2</td>
<td>195.9</td>
<td>200.3</td>
</tr>
<tr>
<td>18</td>
<td>198</td>
<td>203.9</td>
<td>175.3</td>
<td>179.3</td>
</tr>
<tr>
<td>19</td>
<td>197</td>
<td>193.6</td>
<td>183.8</td>
<td>189.2</td>
</tr>
<tr>
<td>20</td>
<td>230</td>
<td>234.6</td>
<td>213.5</td>
<td>216.7</td>
</tr>
<tr>
<td>21</td>
<td>232</td>
<td>229.4</td>
<td>197.3</td>
<td>195.7</td>
</tr>
<tr>
<td>22</td>
<td>226</td>
<td>224.5</td>
<td>179.0</td>
<td>178.1</td>
</tr>
<tr>
<td>23</td>
<td>228</td>
<td>218.4</td>
<td>164.0</td>
<td>159.3</td>
</tr>
<tr>
<td>24</td>
<td>168</td>
<td>157.4</td>
<td>154.8</td>
<td>146.9</td>
</tr>
<tr>
<td>25</td>
<td>144</td>
<td>157.8</td>
<td>121.1</td>
<td>130.5</td>
</tr>
<tr>
<td>26</td>
<td>174</td>
<td>180.6</td>
<td>183.8</td>
<td>189.2</td>
</tr>
<tr>
<td>27</td>
<td>172</td>
<td>168.2</td>
<td>171.7</td>
<td>135.2</td>
</tr>
<tr>
<td>28</td>
<td>192</td>
<td>200.5</td>
<td>195.9</td>
<td>202.6</td>
</tr>
<tr>
<td>29</td>
<td>172</td>
<td>167.2</td>
<td>136.3</td>
<td>135.2</td>
</tr>
<tr>
<td>30</td>
<td>181</td>
<td>166.9</td>
<td>157.0</td>
<td>142.1</td>
</tr>
<tr>
<td>31</td>
<td>190</td>
<td>187.5</td>
<td>175.3</td>
<td>175.1</td>
</tr>
<tr>
<td>32</td>
<td>213</td>
<td>221.4</td>
<td>213.5</td>
<td>219.8</td>
</tr>
<tr>
<td>33</td>
<td>205</td>
<td>199.3</td>
<td>164.0</td>
<td>160.5</td>
</tr>
<tr>
<td>34</td>
<td>209</td>
<td>207.4</td>
<td>179.0</td>
<td>177.9</td>
</tr>
<tr>
<td>35</td>
<td>212</td>
<td>215.3</td>
<td>197.3</td>
<td>199.6</td>
</tr>
</tbody>
</table>

where \( E \) is voltage, \( I \) is current, \( R \) is welding speed, and \( \eta \) is the heat transfer efficiency.

The weld bead reinforcement (\( G \)), defined as the transverse cross-sectional area of the deposited metal, is given by

\[ G = \frac{SD^2 \pi}{4R} \]

(A14)

where \( S \) is the electrode speed and \( d \) is the electrode diameter.

The output voltage of the power supply \( E \) is given by

\[ E = E_o + n I = H \frac{dl}{dt} \]

(A15)

where \( E_o \) is the open circuit voltage of the supply (i.e., at zero load), \( n \) is the effective slope of the supply (i.e., the combined effects of power supply slope and secondary circuit resistance), \( H \) is the power supply secondary circuit inductance, and \( t \) is time.

Current is maintained by the power supply consistent with the contact tube-to-base-metal resistance, which is determined by the electrode extension and
Combining Equations A16 and A17

\[ IE = IV' + IS' \]  \hspace{1cm} (A-16)

where both sides of Equation A-17 are multiplied by the current, and \( R_a \) is the arc resistance. Solving Equation A12 for \( IV' \) and substituting in Equation A-17

\[ IV'_e = I^2R_a \]  \hspace{1cm} (A-17)

where

\[ H_m = \int_{T_e}^{T_i} C_p(T) dT + H_f \]  \hspace{1cm} (A19)

Combining Equations A16 and A18

\[ IE = A\delta S_{m} - nI^2R_a \]  \hspace{1cm} (A18)

Substituting Equation A15 into Equation A20, ignoring the inductance term

\[ I/E = A\delta S_{m} + (1 - \eta')I^2R_a \]  \hspace{1cm} (A20)

and solving for \( I \)

\[ I = \frac{E_i}{\eta n^2 + \eta n + \eta n^2 + \eta n} \]  \hspace{1cm} (A21)

rearranging terms, and combining

\[ [n - (1 - \eta')R_a]T^2 + E_i - A\delta S_{m} = 0 \]  \hspace{1cm} (A22)

and solving for \( I \)

\[ I = \frac{E_i + 4(n - 1 - \eta')R_a}{2[(1 - \eta')R_a - n]} \]  \hspace{1cm} (A23)

Equation A14 is substituted into Equation A13, giving

\[ H_f = \frac{E_i n^2 G}{S} \]  \hspace{1cm} (A24)

Substituting Equation A15 into Equation A24, again neglecting the inductance term, gives

\[ H_f = \frac{E_i n^2 G + n\eta G I^2}{S} \]  \hspace{1cm} (A25)

where \( S \) is equivalent to Equation A23, associated \( n \) and \( c \) are constants and \( C_T \) is contact tube-to-workpiece distance. Using the average \( n^2 \) and associated \( R_a \) values, Equation A23 was

\[ S = \frac{E_i n^2 G + n\eta G I^2}{A H} \]  \hspace{1cm} (A26)

where \( I \) is equivalent to Equation A23, Equation A26 is the same as Equation 10, and

\[ R = \frac{SA}{G} \]  \hspace{1cm} (A27)

Note that Equation A27 is the same as Equation 11, and Equations A23 and A26 are both functions of \( I \) and \( S \).

Two terms in Equation A23 need evaluation, \( \eta' \) and \( R_a \). The first, \( \eta' \), is efficiency of heat transfer from the arc to the electrode, which for our purposes is defined as

\[ \eta' = \frac{dQ}{dt} \frac{EI}{n} \]  \hspace{1cm} (A28)

where \( Q \) is heat transferred from the arc to the electrode.

Results from Waszink and Van den Heuvel (Ref. 3) give

\[ \left( \frac{dQ}{dt} \right) = 6l - 0.7 \times 10^6 m' - 125 \]  \hspace{1cm} (A29)

for an electrode diameter of 1.2 mm, where electrode melting rate is

\[ m' = \frac{8S \pi D^2}{4} \]  \hspace{1cm} (A30)

The electrode diameter used in this work was 0.89 mm, but Equation A29 (which is the same as Equation A11) is used as the best available approximation. Substituting Equation A29 into A28 gives

\[ \eta' = \frac{6l - 3415S - 125}{(E_i + n^2)} \]  \hspace{1cm} (A30)

For conditions used in this work, values of \( \eta' \) have been calculated and listed in Table 1.

A statistical analysis program, SAS (Ref. 15), was used to determine the arc resistance \( R_a \) values which give the best agreement between actual and calculated currents; values range between 0.10 and 0.15 Ω. In addition, the best-fit \( R_a \) values were obtained for \( \eta' \) = 0.092, this fit being of the functional form:

\[ R_a = K_0 + K_1 (CT) + K_2 E + K_3 I + K_4 (CT) E \]  \hspace{1cm} (A31)

where \( K_0, K_1, K_2, K_3, \) and \( K_4 \) are constants and \( CT \) is contact tube-to-workpiece distance. Using the average \( \eta' \) and associated \( R_a \) values, Equation A23 was
7 = CT0 + tS

The calculated current values are listed in Table 2. The standard deviation of the calculated current was ±7.5 A, for the electrode speed it was ±3.5 mm/s. However, certain welds resulted in negative values for the quadratic term in Equation A23. Arbitrarily eliminating these negative values gave a standard deviation of current of ±5.8 A, and for electrode speed of ±3.9 mm/s.

Calculation of Equation (A23) proved to be too time consuming for controlling the welding hardware with the software used, which is in an interpretive language. To overcome this problem, the liberty was taken of using a SAS-derived (Ref. 15) equation for current:

\[
I = K'0 + K'_1(CT) + K'_2S \tag{A-32}
\]

where the K' terms are again constants. Equation A32 is the same as Equation 12. Results of using this equation in place of Equation A23 are shown in Table 3. Equation A32 is slightly less accurate than Equation A23 in predicting current.

Equation A26 is of the same functional form as 1, and Waszink and Van den Heuvel's similar Equation 37 (our Equation 2). It may be noted that the functional form of Equation A26 is governed by the substitution of Equation A15 into Equation A24. That is, the order of Equation A26 in I is determined by the product of I times the governing equation for the power supply. If the power supply is linear in I, the heat input to the weld will be quadratic in I.

Recent work by Kim, et al. (Ref. 16), has shown that there are conditions under which the quadratic function form discussed above does not hold. Specifically, the anode spot on the liquid drop on the electrode wire end may envelop the drop, extending onto the wire above the drop. This results in an offset or jog in the melting rate vs. current curve. We have not attempted to include this additional term in our analysis, but rely on the controller to compensate for its occurrence.

Appendix B

Terminology

- \( m_e \): Electrode melting rate
- \( A \): Electrode cross-sectional area
- \( I \): Calculated current

\( \eta' \): Arc to electrode heat transfer efficiency

\( V_a \): Voltage drop across arc

\( \delta \): Electrode feed speed

\( H_m \): Total heat required for a unit volume of material to go from \( T_p \) to \( T_0 \)

\( T_0 \): Ambient temperature

\( H \): Heat input per unit length of weld to base metal

\( G \): Secondary circuit voltage drop

\( E_0 \): Process to base metal heat transfer efficiency

\( R_a \): Weld bead reinforcement

\( Q \): Power supply open circuit voltage

\( n \): Effective power supply slope

\( K_0 \): Secondary circuit inductance

\( T \): Time

\( K_i \): Sink temperature

\( K' \): Constant

\( K_{p-1} \): Controller proportional gain

\( e(t) \): Error signal input to controller as function of time

\( K_{i} \): Controller integral gain

\( U \): Measured, filtered current = \( I \)

\( W \): Internal energy

\( x \): Work

\( \rho \): Electrical resistivity

\( \sigma \): Thermal emissivity

\( \kappa \): Stefan-Boltzmann constant

\( T_m \): Melting temperature

\( \phi^*, H^*, Q^* \): Constants

References


