A Study of the Thermal Behavior in Resistance Spot Welds

Prediction models for temperature distribution and nugget growth compare well with actual spot weld experiments

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ABSTRACT. Due to the complexities arising from the flow paths of electric current, the material property variations with temperature, and the phase change problem, analytical prediction of the thermal behaviors in resistance spot weld has not been fully attempted. This paper presents a theoretical basis to this problem, taking into account such complexities. Finite difference models are developed to predict the temperature and voltage distributions during the nugget formation, incorporating the thermoelectric interaction at the interface in the weldment. The computed results based upon these models are compared with experimentally obtained ones for the case of welding mild steel. The comparison result shows that diameter growth and penetration of the nugget, as well as the temperature field in the weldment, are highly predictable.

Introduction

Resistance spot welding is a process that has been widely used in sheet metal fabrication, with the advantages of high speed, suitability for automation and inclusion in high-production assembly lines with other fabricating operations. In this welding process, the facing surfaces of two or more workpieces are fused and joined in spot by Joule's heat due to the flow of electric current through a weldment held together under electrode force (Ref. 1). Due to the complexity of the process, an adequate understanding of thermal behavior in the weldment, based upon theoretical analysis, has been a difficult task. The complexity arises from the interaction of many factors such as the complex flow paths for heat and electrical current and the variation in the properties of the materials with temperature and phase change. Therefore, only a few results have been published for analytical studies, while innumerable studies (Ref. 2) to investigate the weldability and thermal behavior for various metals have been performed experimentally, based upon the observation of metallurgical structure.

In 1960, Archer (Ref. 3) showed the effect of variation in welding time and the phase change control of heat input on temperature distribution for some thin alloy sheet metals, assuming constant electrical current and the variation in the material property variations with temperature. Therefore, only a few results have been published for analytical studies, while innumerable studies (Ref. 2) to investigate the weldability and thermal behavior for various metals have been performed experimentally, based upon the observation of metallurgical structure. The interfacial resistance in the resistance spot weld was introduced by Holm (Ref. 6). More practical contact behavior for various load conditions were extensively investigated by Nied (Ref. 7), including the nonlinear thermomechanical coupling. In his investigation, an ANSYS finite element program was used to construct the process model. Though his analysis allows prediction of the deformation of the welding electrodes and workpieces, the physical phenomena at the interface are not considered in the analysis. Furthermore, since the ANSYS program is not developed primarily for the welding process, analysis requires a very large computing capability and quite a lot of knowledge to implement.

The aim of the present study is to provide an analytical tool that can handle physical properties that have not been considered in the previous studies. To this end, a finite difference model is established, which is simple and easy to implement with a microcomputer. This model contains the following major considerations. Firstly, the internal heat generation due to the passage of electric current through base metal and the interface of weldment is included for the analysis. Secondly, the complicated ther-
moelectric interaction at the interface in the weldment is rigorously treated. Thirdly, the phase change problem from solid to liquid is considered, taking into account growth of nugget geometry.

Simulations were performed to predict the time behavior of the temperature and voltage distribution in the weldment for various heat inputs. Also, the variation of nugget geometry with welding time is predicted. Inclusion of those above-mentioned aspects in the simulations, however, leads to the computation complexity. To make this easier, two finite difference schemes were employed. The first is an Alternating Direction Implicit (ADI) method (Ref. 10), which was used to obtain the unsteady temperature field in the weldment, and the second is the Successive Overrelaxation (SOR) method (Ref. 11), used to obtain the voltage distribution in the weldment. The simulation results thus obtained were compared with those of the experiments. These comparisons show that despite several assumptions made in the theory, the predicted results are in good agreement with those experimentally obtained.

Formulation of the Problem

The resistance spot welding mechanism is illustrated schematically in Fig. 1. In the welding, the required heat is generated by a passage of a high-current pulse across the welding interface and base metal. Under the mechanical and electric loading conditions in the weldment, the voltage potential field is formed within the base metal and along the electrode and weldment interface. This voltage distribution causes a current flow as follows (Ref. 16):

\[ J = \frac{1}{\sigma} \nabla \phi \]

where \( J \) is the current density vector, \( \sigma \) is the resistivity of the welded material and \( \phi \) is the voltage potential. When the current density in the weldment is high enough, the melting initiates at the interface of the two workpieces. To prevent the electrode from sticking to the weldment, part of the generated heat in the electrode interface is dissipated through the copper alloy of the electrode to cooling water.

Once melting is started, the contacts move closer together through a squeezing action by the electrodes, enlarging the area of the contact. Thus, contact resistance is altered, modifying in turn the distribution of voltage field in the weldment. As a result, current density will be redistributed in the weldment and a new temperature distribution in the weld is established. This procedure can be summarized as follows:

1) The material properties are given and the weldment is squeezed by two electrodes.
2) Electrical loading is self-adjusted by the welding system impedance.
3) Voltage potential field is formed within the weldment in agreement with the electrical loading.
4) Temperature is appropriately distributed in the weldment and the electrode, balanced by the heat generation of current flow and heat loss to the cooling water.
5) The interface between the weldments is collapsed due to the temperature rise, and the overall lumped resistance varies with the temperature distribution in the weldment and the nugget geometry.

The above procedure is repeated as many times as needed during the welding process. The subsequent theoretical analysis is performed to obtain a solution to temperature and voltage distribution in the weldment.

Simplification of the Behavior of the Interface in the Weldment

The interface of the weldment is characterized by a certain roughness and waviness. These characteristic features cause surfaces in contact to rest on their asperities forming microcontact points. The convergence of the electric current flow lines toward these microcontact points results in an electrical constriction resistance. This constriction resistance at the interface causes a voltage drop across it and generates heat to fuse the workpieces.

The electrical resistance at the interface changes greatly during the welding process and depends on the mechanical properties of the material adjacent to the interface, the amount and kind of impurities, the apparent pressure and the temperature distribution, which strongly affect the mechanical properties (Refs. 6, 8).

With the complicated thermoelectric behavior of the joint taken into consideration, along with the various phenomena and the thermal and electrical boundary conditions imposed on the problem, a determination of optimum characteristics at the interface is required to deal with the variety of interactions experienced with a joint of this type. As current flows across the interface, voltage distribution in a plane normal to the interface seems to be discontinuous. In reality, this is not true, and the sharp voltage drop between extrapolated voltage values on either side of the interface is a "fictitious interface voltage drop" as shown in Fig. 2. It makes sense, therefore, to assume linear variation in voltage within the disturbed zone, thus maintaining a continuity in the voltage potential field. Then, idealization of the interface of electrical current flow can be done by assigning an averaged contact resistance, \( \sigma_c \), from the microscopic point of view and the current density across the interface of unit area, \( J \), is given by

\[ J = \frac{1}{\sigma} \frac{\partial \phi}{n} = \frac{1}{\sigma_c} \Delta \phi \]

where \( \sigma_c \) is the interfacial resistivity, \( n \) is the normal vector to interface plane and \( \Delta \phi \) is the voltage drop at the interface. In the above, the averaged contact resis-
Voltage Potential Field

The voltage potential field drop in the electrode can be neglected, following assumptions: 1) the voltage distribution is not only used to calculate the heat generation term as follows:

\[ \rho_w \sigma_w \frac{dT_w}{dr} = \frac{\partial}{\partial r} (k_w \frac{dT_w}{dr}) + \rho_w \sigma_w \frac{dT_w}{r} + \frac{1}{\sigma} \nabla \phi \cdot \nabla \phi \]

and the temperature distribution in the electrode, \( T_e \) is

\[ \rho_w C_w \frac{dT_e}{dr} = k_w \frac{dT_e}{dr} + \frac{\partial^2 T_e}{\partial z^2} \]

and the boundary conditions are from the geometric symmetry

\[ \frac{\partial}{\partial r} T_w(r, z) = \frac{\partial}{\partial r} T_e(r, z) = 0 \quad 0 \leq z \leq D_2 \]

from the assumption (3) and at the contact surface from Equation 4

\[ -k_w \frac{\partial}{\partial z} T_w(r, 0) = \frac{1}{\sigma_w} \phi \quad 0 \leq r \leq R_2 \]

\[ -k_w \frac{\partial}{\partial z} T_w(r, 0) + \frac{1}{\sigma_e} (\phi - \phi_e)^2 = -k_e \frac{\partial}{\partial z} T_e(r, 0) \quad 0 \leq r \leq R_1 \]

and on the water-cooled surface of the electrode

\[ -k_e \frac{\partial}{\partial z} T_e(r, 0) = h_w [T_e(r, D_2) - T_{co}] \]

Temperature Distribution in the Weldment

In describing the thermal behavior in the weldment and electrode, some assumptions precede the precise analysis:

1) The thermal conductivity of the electrode made of a copper alloy is assumed to be constant.
2) The pressure distribution is assumed to be uniform at the interface between the two workpieces and at the electrode-to-workpiece interface.
3) Free convection heat loss at the surface, except at the contact interface, is assumed to be very small, compared with the heat loss due to the cooling water. Then, the temperature distribution, \( T_w \), in the weldment is described as an unsteady heat conduction problem with heat generation term as follows:

\[ \rho_w C_w \frac{dT_w}{dr} = \frac{\partial}{\partial r} (k_w \frac{dT_w}{dr}) + \rho_w \sigma_w \frac{dT_w}{r} + \frac{1}{\sigma} \nabla \phi \cdot \nabla \phi \]

Voltage Potential Field

In this section, the voltage distribution in the weldment is examined, relying on electrical loading, temperature distribution and nugget geometry. The voltage distribution is not only used to calculate the heat generation in the weldment for an analysis of thermal behavior, but also to obtain the instantaneous variation of resistance of the weldment under the following assumptions: 1) the voltage drop in the electrode can be neglected, compared with that in the weldment, since the electrical conductivity of the electrode is much greater than that of the weldment, and 2) the magnetic stirring effect (Ref. 10) in the nugget is not considered. The subsequent quasi Laplace equation and the boundary conditions represent the voltage potential, \( \phi \), i.e.,

\[ \frac{\partial}{\partial r} \left( \frac{\partial \phi}{\partial r} \right) + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad 0 < r < R_1 \]

and the boundary conditions, except at the contact surface, are given as insulated, i.e.,

\[ \frac{\partial}{\partial r} \phi(r, z) = \frac{\partial}{\partial r} \phi(r, D_1, z) = 0 \quad 0 \leq r \leq D_1 \]

\[ \frac{\partial}{\partial z} \phi(r, 0) = \frac{\partial}{\partial z} \phi(r, D_2, 0) = 0 \quad 0 \leq z \leq D_2 \]

Fig. 2 — Model of contact surface

\[ \sigma_c = \sqrt{\frac{H(T)}{H(T_{co})}} \sigma_c(T_{co}) \]

where \( H \) is the hardness, \( T \) is the temperature and \( \sigma_c(T_{co}) \) is the interfacial resistivity at room temperature, \( T_{co} \). The hardness is a decreasing function of temperature, and the variation of hardness for mild steel and copper is illustrated in Ref. 9. Then, the heat generation at the interface, \( q \), becomes

\[ q = \frac{1}{\sigma_c} \Delta \phi^2 \]

Thickness

not to scale

\[ J = \frac{1}{\sigma_c} \Delta \phi \]

Voltage

Frictional voltage drop

Voltage drop

Disturbed zone

Thickness

Disturbed zone

not to scale

Voltage drop
Table 1—Chemical Composition of the Weld Metal (wt-%)

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19-0.21</td>
<td>0.62</td>
<td>0.043</td>
<td>0.012</td>
<td>0.018</td>
<td></td>
</tr>
</tbody>
</table>

where \( \rho, C \) and \( k \) are density, specific heat and thermal conductivity, respectively, \( V \) is the gradient of voltage potential, \( T_{cool} \) is the cooling water temperature (\( 20°C/68°F \)) and the other geometric parameters such as \( D_1, D_2, R_1 \) and \( R_2 \) are given in Fig. 1. In the above equations, the subscripts \( w \) and \( e \) denote the weldment and the electrode, respectively.

### Numerical Solution Method

#### Finite Difference Equation

In order to obtain the temperature and the voltage distribution in the weldment numerically, the governing Equations 5, 7 and 8, and the corresponding boundary conditions are transformed into appropriate finite difference equations.

The finite difference equation for Equation 5, which represents voltage distribution in the weldment, is given by

\[
\frac{1}{\Delta r^2} \left(\Phi_{w}^{i+1,j} - 2\Phi_{w}^{i,j} + \Phi_{w}^{i-1,j}\right) - \frac{1}{\Delta r^2} \left(\Phi_{e}^{i+1,j} - 2\Phi_{e}^{i,j} + \Phi_{e}^{i-1,j}\right) = -w_{ijkl}^* \Phi_{ijkl}^{i,j} + \Phi_{ijkl}^{i,j+1} + \Phi_{ijkl}^{i,j-1}
\]

where \( \sigma_{ij} \) is a local resistivity dependent on local temperature in the weldment and \( \Delta r \) and \( \Delta z \) are the step sizes in radial and thickness directions, respectively.

The solution is obtained by successive overrelaxation or SOR method (Ref. 11) with an appropriate relaxation factor, \( w (1 < w < 2) \). The \( \Phi_{ijkl}^{i,j} \) is calculated iteratively from the transformed equation into a relaxation form as follows:

\[
(I - wL)\Phi_{ijkl}^{k+l} = wb + [(1 - w)L + wU] \Phi_{ijkl}^{k}
\]

where \( k \) is an iteration number, \( L \) and \( U \) one lower and upper triangular matrix including coefficients and \( b \) is a coefficient vector.

In analyzing the unsteady temperature distribution in the weldment and electrode, the D’Yakonov formula (Ref. 11),

\[
\delta (\Theta_{ijkl}^{i,j}) = \frac{1}{2} \left[ (\Theta_{ijkl}^{i+1,j} + \Theta_{ijkl}^{i,j-1} - \Theta_{ijkl}^{i,j+1} - \Theta_{ijkl}^{i,j-1}) \right]
\]

which is a kind of alternating direction implicit method (ADI), is employed. Its merits of numerical stability and accuracy are fairly well explained in Ref. 11. The finite difference equations for Equations 7 and 8 can be written as

\[
[1 - \frac{\alpha_w}{\Delta t} (\frac{\delta r^2}{2} + \frac{\delta z^2}{2})] \Theta_{ijkl}^{i,j} = \left[ \left(1 + \frac{\alpha_w}{\Delta t} (\frac{\delta r^2}{2} + \frac{\delta z^2}{2}) \right)\Theta_{ijkl}^{i,j} \right]_{t^{k+1}} - \left[ \left(1 + \frac{\alpha_w}{\Delta t} (\frac{\delta r^2}{2} + \frac{\delta z^2}{2}) \right)\Theta_{ijkl}^{i,j} \right]_{t^k}
\]

and \( \alpha_w = \frac{\Delta t}{\rho_{c} C_{w}} \), \( \alpha_e = \frac{\Delta t}{\rho_{e} C_{e}} \) and \( k_{ij}^* \) is the thermal conductivity corresponding to the temperature \( \Theta_{ijkl}^{i,j} \).

### Simulation

Having obtained the complete finite difference equations, we can proceed with the numerical study on the behavior of the temperature and voltage distribution in the weldment. In this simulation, the time behavior of these distributions will be numerically obtained and the effect of heat input will also be investigated. For the simulation study, a mild steel plate of thickness \( 1.6 \text{ mm (0.06 in.)} \) is used and its chemical composition is shown in Table 1.

The difference grid geometry used for the numerical calculation is depicted in Fig. 3. The mesh size, \( \Delta r \), is 0.2 mm, \( \Delta z \) is 0.1 mm for the weldment and 0.5 mm for the electrode. The domain concerned in

![Fig. 3 — Grid for numerical calculation](image_url)
The weldment is limited to a cylinder with a height of 1.6 mm and a diameter twice as large as that of the electrode contact area. The time step size, \( \Delta t \), was taken as a quarter of a half-cycle period of a 60-Hz AC power supply (about 2 ms). The ADI method employed in this problem is unconditionally stable (Ref. 11), but for a larger time step size than the above value, some oscillation in temperature response of the weldment occurred near the contact surface. In calculating the voltage distribution with the SOR method, the overrelaxation factor makes an important role for converging speed of solution. In this study, it was fixed as 1.85 after some trial and error.

The electrical resistivity, thermal conductivity and specific heat of mild steel (Table 2), which are contained in Equations 5 and 7, are known to vary considerably with temperature (Ref. 12). In order to incorporate these variations into the simulation, those parameters were cited from the table in the ASM Metals Handbook (Ref. 12) and also from the polynomial equations as given in Ref. 5. The values of the contact resistivity of the weldment and the electrode at the room temperature \( \sigma_{cw}(T_0) \) and \( \sigma_{ce}(T_0) \) were obtained from Ref. 13.

For phase change problem, the latent heat effect was included by using the fractions of specific heat increase (Ref. 14) at the temperature between solidus and liquidus. That is, the specific heat, \( C_w \), is approximated as

\[
C_w = \frac{H_f}{T_f - T_s}
\]

In the above equation, \( H_f \) is the latent heat of fusion, \( T_f \) and \( T_s \) are the solidus and liquidus temperatures, respectively.

In calculating the voltage distribution in the weldment, the voltage drop between the electrode and the interface of weldment, \( \phi_{e} \), has to be a priori known to be substituted into the boundary condition (Equation 6.4). It is a continuously varying value from cycle to cycle according to the heat control and the welding system impedance. The typical response of voltage \( \phi_{e} \) is illustrated in Fig. 4 for various currents, which are the averaged value for every half-cycle measured by a weld checker (Model MM-502A, Miyachi Electronic Co.). Figure 5 shows the contribution of three different parts in the weldment to overall voltage drop, \( \phi_{e} \), along the axis \( r = 0 \): the electrode interface, the base metal and the weldment interface. The voltage drop contribution was obtained for the case of the welding current, 6.1 kA, which is shown in Fig. 4C.

In the beginning of the welding, the electrode interface takes place in the interface between electrode and weldment, and in the interface of the two workpieces (21% and 66% respectively), i.e., the resistance in the workpiece interface is much greater than the base metal resistance (Ref. 1). This demonstrates the importance of the interface resistance in the early stage of spot welding. As time increases, the ratio of the voltage drop at the interface to the overall voltage drop tends to decrease, on the other hand, the voltage drop ratio in the base metal increases. When the welding time reaches six cycles from the instant that the fusion of the weldment begins at its center, the overall voltage drop almost consists of that in the base metal (87%). This is due to the fact that the interfacial resistance decreases as temperature rises, while the resistivity of the base metal increases.

The simulations were performed for various heat inputs, taking into account the above considerations, and the results are discussed in detail in a later section.

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**Table 2—Physical Properties of the Weldment and Electrode**

<table>
<thead>
<tr>
<th>Property</th>
<th>Notation (Unit)</th>
<th>Weldment (Mild Steel)</th>
<th>Electrode (Copper Alloy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>( k/(W/m \cdot ^\circ C) )</td>
<td>( k_w = k_w(T_{eq}) )</td>
<td>( 3.014 \times 10^3 )</td>
</tr>
<tr>
<td>Electrical resistivity</td>
<td>( \sigma_{d}(\mu \Omega-m) )</td>
<td>( \sigma_w = \sigma_w(T_{eq}) )</td>
<td>( 3.83 \times 10^{-2} )</td>
</tr>
<tr>
<td>Specific heat</td>
<td>( C/(J/kg \cdot ^\circ C) )</td>
<td>( C_w = C_w(T_{eq}) )</td>
<td>( 4.102 \times 10^2 )</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho/(Kg/m^3) )</td>
<td>( 7.86 \times 10^3 )</td>
<td>( 8.93 \times 10^3 )</td>
</tr>
<tr>
<td>Latent heat of fusion</td>
<td>( H_f/(J/kg) )</td>
<td>( 2.742 \times 10^3 )</td>
<td>-</td>
</tr>
<tr>
<td>Convection heat coefficient</td>
<td>-</td>
<td>-</td>
<td>( 4.187 \times 10^4 )</td>
</tr>
<tr>
<td>Solidus temperature</td>
<td>( T_*{T_f}(^\circ C) )</td>
<td>1493</td>
<td>-</td>
</tr>
<tr>
<td>Liquidus temperature</td>
<td>( T_{eq}(^\circ C) )</td>
<td>1525</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) Refs. 5, 12.
Experiments

The objective of these experiments was to evaluate the previous simulation results of the variation in temperature and voltage distributions in the weldment. The experiments were performed under the welding conditions corresponding to those of the simulations.

Experimental Apparatus

The experiment was carried out on an air-operated resistance spot welding machine (Model SP-AF50, Cho-Heung Welding Co.) of which the rated capacity is 50 kVA. The heat input was controlled by adjusting the SCR's firing angle. The squeeze time and the hold time were set as recommended by the Resistance Welding Manufacturers Association (RWMA). The electrode used was a RWMA Class II with a tapered flat shape and a 5.5-mm (0.22-in.) diameter contact surface. During the welding process, the variation of voltage, current, and resistance were monitored using the weld checker. This device displays simultaneously the averaged values of the above three variables for every half cycle.

Preparation of Specimens

The weldments used in this experiment were the same as those used in the simulation. As a surface treatment prior to welding, the workpieces were degreased with toluene to maintain a uniform surface condition. After welding, each specimen was subject to cross-sectioning along the centerline of the nugget, mounting, polishing and macro-and microetching with 5% Nital solution.

Experimental Results

Figure 6 shows a typical macro- and microstructure in the cross-section of the specimen. It can be observed in the macrophotograph that the weldment consists of three different structures. The first, which occupies the middle of the weldment, is a dendritic structure that appears in the nugget when the molten metal is solidified (Ref. 15). The next region encircling the nugget is the heat-affected zone (HAZ) where the austenite is partially transformed into martensite that is developed with rapid cooling from above the A1 transformation point. The metallurgical structure beyond the HAZ remains unaffected by heat. Based upon the above statements, one can obtain at least two isothermal lines (the solidus temperature and the A1 transformation temperature) on the macrophotograph of the prepared specimens.

In order to study the nugget formation process during welding, some workpieces were welded, keeping the average welding current at 6.1 kA and the electrode force at 360 kgf, and increasing the total welding cycles from 8 to 16. The macrophotographs obtained under these conditions are illustrated in Fig. 7. Another set of specimens was prepared to investigate the effect of magnitude of current on the temperature distribution. In this case, the total welding cycle was fixed at 16 cycles, and the electrode force at 360 kgf. Figure 8 shows the results of the experiments. While the second and third welds can be accepted as good, the first shows an insufficient nugget size, and the last weld is about to start an expulsion (two workpieces are separated significantly and the melt bursts out from the left).

Comparison between Theory and Experiment

Temperature Response with Time Lapse

The variations of temperature distribution in the weldment versus the welding time are illustrated in Fig. 9. The left-hand side of the figures represents the experimental results, which are illustrated in Fig. 7, while the right-hand side denotes the theoretical results based upon the numerical calculation. It can be observed that the measured and the calculated curves of the isothermal lines at 1493°C (2719°F) (the boundary of the nugget) coincide very well, though the nugget diameter in the calculation is smaller than that in the experiment. But the isothermal
Fig. 8—Macrophotographs of cross-sectioned specimens showing the variation of nugget dimension versus heat input: welding time = 16 cycles, electrode force = 360 kgf

lines at 723°C (1333°F) show some differences in shape and size. The curves at 723°C in the calculation tend to occupy a larger area, thus, the HAZ spreads wider compared with those in the photographs. This is probably due to the fact that the deflection of the weldment by the electrode was neglected in the calculation. In the real process, the two workpieces are distorted outward by electrode squeeze, and thus, the heat generation is concentrated on the center of the nugget, since the current density in the center of the nugget is much higher than that of the outer electrode contacted part. This phenomena, which was not observed in Yamamoto's study (Ref. 5), is expected to appear in the welding of light-gauge steel plate, since it is easily deflected by a small electrode force.

The calculated results show that the temperature near the edge of the electrode contact is very high, compared with the measured one, which is due to a great heat generation at the edge of the contact. This can be explained by the observation of the surface of the weldment after welding. In actual welding, the edge of the electrode becomes soft and rounded due to the adjacent high temperature, which causes a small current density near the edge, compared with the case of the calculation in which a sharp edge contact is assumed through all of the welding cycle.

The calculated results show that the mean temperature gradient along with the thickness is about -811.87°C/mm, which is approximately three times as great as that along the radial direction, -250.37°C/mm. Thus, of the generated heat in the weldment, the heat flow along the thickness escapes through the electrode to the water rapidly, while the heat flow along the radial direction contributes to the increase of nugget diameter.

In Fig. 13, the effect of the heat input
on the variation of nugget size is depicted quantitatively for both cases of the experiment and the calculation. The nugget penetration growth remains constant regardless of the mean current value. On the other hand, the nugget diameter increases gradually as the mean current increases. The square of nugget diameter is almost proportional to the square of the mean current with the correlation of 95%. The above fact is maintained as identical in the numerical calculation except for the fact that the nugget diameter in the experiment tends to be greater than that in the calculation. According to Joule's law, the rate of heat generation in the weldment is proportional to the square of the welding current, thus it can be explained that the nugget volume at the end of welding increases proportional to the rate of overall heat generation.

**Conclusions**

An analytical thermoelectric model has been developed to predict growth of the nugget geometry, as well as to analyze the temperature distribution in the weldment. It takes into account the effect of
the thermoelectric interaction at the weldment interface on the internal heat generation, which has been a major hurdle in spot welding thermal analysis. Based upon this model of time behavior of the temperature field in the weldment, nugget diameter growth and nugget penetration have been numerically obtained via ADI and SOR schemes. To verify these analytical results, a series of experiments was performed under the same welding conditions used in the analysis. Both results are generally in good agreement, although prediction of the heat-affected zone (HAZ) appears to require some refining work in the theory. Furthermore, the implication of these results is that the proposed analytical model and numerical scheme can replace the present trial and error method in establishing welding conditions needed for certain weld materials.

References

Appendix
Nomenclature

\[ C_e = \text{specific heat of electrode} \]
\[ C_w = \text{specific heat of weldment} \]
\[ D_1 = \text{thickness of weldment} \]
\[ D_2 = \text{distance from the center of weldments to water-cooled surface} \]
\[ h_w = \text{convection heat coefficient to water} \]
\[ H = \text{hardness of material} \]
\[ H_F = \text{latent heat of fusion} \]
\[ i = \text{node number in radial direction} \]
\[ j = \text{node number in thickness direction} \]
\[ L = \text{current density vector} \]
\[ R_e = \text{thermal conductivity of electrode} \]
\[ k_w = \text{thermal conductivity of weldment} \]
\[ n = \text{normal vector to interface plane} \]
\[ q = \text{heat generation at interface} \]
\[ R_1 = \text{radius of electrode contact surface} \]
\[ R_2 = \text{radius of weldment included in numerical analysis} \]
\[ \Delta r = \text{grid size in radial direction} \]
\[ \Delta t = \text{time increment in numerical analysis} \]
\[ T_e = \text{temperature of electrode} \]
\[ T_L = \text{liquidus temperature of weldment} \]
\[ T_s = \text{solidus temperature of weldment} \]
\[ T_w = \text{temperature of weldment} \]
\[ T_{co} = \text{temperature of cooling water} \]
\[ \sigma = \text{grid size in thickness direction} \]
\[ \delta_e = \text{difference operator} \]
\[ \delta_e^* = \text{approximation of } \delta_e \text{ in numerical analysis} \]
\[ \delta_e^{**} = \text{intermediate value of } \delta_e \]
\[ \delta_w = \text{approximation of } \delta_w \text{ in numerical analysis} \]
\[ \delta_w^{**} = \text{intermediate value of } \delta_w \]
\[ \rho_e = \text{density of electrode material} \]
\[ \rho_w = \text{density of weldment material} \]
\[ \sigma_e = \text{resistivity of the material} \]
\[ \sigma_e = \text{interfacial resistivity of the material} \]
\[ \sigma_w = \text{interfacial resistivity in electrode and weldment contact} \]
\[ \phi = \text{voltage potential} \]
\[ \phi_e = \text{voltage drop between electrode and the center of weldments} \]
\[ \phi = \text{approximation of } \phi \text{ in numerical analysis} \]

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Review of Properties of Thermo-Mechanically Controlled Processed Steels—Pressure Vessel Steels for Low-Temperature Service

Japanese steelmakers have developed the Thermo-Mechanical Control Process (TMCP) that includes an accelerated cooling process in the plate mill. Fabricators have utilized various highly efficient welding technologies in their fabrication. Accordingly, a great deal of joint work has been carried out to put this steel and welding technology into practical use. This report summarizes the development of TMCP steel in Japan and was prepared by their Subcommittee on Pressure Vessel Steels.

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